

# A Particle Filter Approach for InSAR Phase Filtering and Unwrapping

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**Abstract**—This paper presents a new phase-unwrapping (PU) algorithm for SAR interferometry that makes use of a particle filter (PF) to perform simultaneously noise filtering and PU. The formulation of this technique provides independence from noise statistics and is not constrained by the nonlinearity of the problem. In addition, an enhanced variant of this method combining a PF with artificial-intelligence search strategies and an omnidirectional local phase estimator, based on the mode of the power spectral density, is also presented. Results obtained with synthetic and real data show a significant improvement with respect to other conventional unwrapping algorithms in some situations.

**Index Terms**—Artificial intelligence (AI), extended Kalman filter (EKF), grid filter, particle filter (PF), path following, phase unwrapping (PU), SAR interferometry, state space.

## I. INTRODUCTION

ONE OF THE key processing steps in all SAR interferometry applications is phase unwrapping (PU). A lot of literature works concerning different techniques to solve the PU problem have been published during the last years. We refer to the book by Ghiglia and Pritt [1] for an excellent overview. Traditionally, there have been two general types of conventional PU methods. The algorithms of the first group, generally named path-following or region-growing algorithms, isolate and/or mask problematic zones containing residues (we refer to [1] for the definition of residues) and unwrap the interferogram by avoiding these zones. The network cost flow algorithm by Costantini *et al.* [2] can be regarded as an extension of the classical branch-cut methods contained in this first group. The techniques of the second group provide a global solution which minimizes a cost function over the whole interferogram. Independently from this traditional classification, some techniques make use of a prefiltering stage before starting the unwrapping procedure with the filtered phase (for instance, [3]–[8]). Consequences of all these strategies are the following: Phase information in noisy pixels is not recovered by the path-following algorithms; noisy pixels distort the solution in global approaches, thus affecting the noise-free areas; and the information contained in noisy pixels is lost if a prefiltering stage is carried out.

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During the last years, a complementary group of approaches has appeared, known as “multichannel” techniques, which require multiple acquisitions (not only two images) for being applied. See, for instance, solutions presented in [9]–[13]. Multichannel techniques have the advantage to eliminate or at least mitigate the ambiguity problem related to the wrapping operator. Note that the approach proposed in this paper is devised for conventional “single-channel” interferometry, i.e., for a single interferogram obtained with only one pair of images.

In a Bayesian framework, different PU techniques are analyzed in [14], and a stochastic nonlinear filtering approach is also presented in [15]. However, Wishart nature of the noise present in the model observation is not considered, and results involving real data are not shown.

From our point of view, the key idea is to recover as much information as possible from the interferogram, including noisy pixels. Therefore, an ideal method would consider every pixel to be simultaneously unwrapped and filtered.

There exists an algorithm that shares the same objective and was already published some years ago in [16]–[20]. This approach, recently revisited in [21], combines a local phase estimator with an extended Kalman filter (EKF) in an elegant fashion to simultaneously achieve noise reduction and PU. Unfortunately, that solution is founded on the following two assumptions. First, both evolution model and measurement model are available and correspond to linear functions. Second, the noise affecting both evolution and measurement stages is Gaussian. When the constraint of Gaussian noise holds but the models are not linear functions, the EKF has been used in many cases as a good solution that approximates the nonlinearities by local linearizations. The main problem arises when the noise present in the interferogram is not Gaussian. The EKF cannot guarantee a successful PU, as we will show later with some examples. Note that a detailed tutorial about different methods to broach nonlinear/non-Gaussian problems can be consulted in [22].

In a previous work, we proposed to substitute the EKF by a grid-based filter (GbF), which is not subject to any linear or Gaussian constraints, and a good performance was demonstrated with both simulated [23] and real data [24].

In this paper, we introduce a particle-filter phase-unwrapping (PFPU) algorithm. We will show, with some examples from synthetic and real data, that this solution also performs as the GbF, but with the great advantage of a lower computational cost. The proposed method integrates a particle filter (PF) and a local phase estimator to simultaneously unwrap and filter the interferometric phase. In addition, this approach can be combined with path-following strategies to enhance their

performance with the embedded noise filtering. We present first the algorithm without any conditioned scheme for choosing the unwrapping path, showing its good performance when dealing with residues. In particular, the basic algorithm introduced in this paper runs the interferogram row by row, passing through zones of low coherence and containing residues, unwrapping and filtering the phase at the same time.

There exist also scenes that include zones of high density of residues or low coherence, which would be preferable not to unwrap until more favorable areas have already been unwrapped. Consequently, a better approach would incorporate some strategy to unwrap the best pixels first and the worst ones at the end. To broach this kind of situations, we have also developed a second improved version of the PFPU algorithm, the so-called 2PFPU, by combining a PF with an enhanced omnidirectional variant of the local phase estimator and a path-following method, based on artificial-intelligence (AI) search strategies. We refer to the book by Nilsson [25] for an excellent overview on AI search strategies.

This paper is organized as follows. Section II introduces the basic formulation of the PU problem. The PF formulation is presented in Section III. The application of PFPU and its improved version (2PFPU) is presented in Section IV. Then, Section V illustrates the performance of the new methods with some examples from both a synthetic data set extracted from [1] and real interferograms. A comparison with some conventional algorithms, namely, Goldstein branch-cut, Flynn's, and preconditioned conjugate gradient (PCG) algorithms [1], the cited EKF approach [16]–[21] and a GbF [23], [24] solution, is also discussed. Finally, conclusions are drawn in Section VI.

## II. PU BASIC FORMULATION

Assuming a 1-D notation, the complex coherence between two SAR images  $\gamma$  is, at pixel  $k$

$$\gamma(k) = a(k) \cdot \exp[j \cdot \tilde{\varphi}(k)] \quad (1)$$

where  $a(k)$  is the observed interferometric coherence (between 0 and 1) and  $\tilde{\varphi}(k)$  is the modulo  $2\pi$  mapped interferometric phase, also called the wrapped phase. This phase mapping can be expressed by

$$\tilde{\varphi}(k) = [\varphi(k) + \tilde{e}_\varphi(k)]_{|2\pi} = \varphi(k) + \tilde{e}_\varphi(k) \pm n \cdot 2\pi \in (-\pi, \pi] \quad (2)$$

where  $\varphi(k)$  is the true unambiguous absolute phase at pixel  $k$  and  $\tilde{e}_\varphi(k)$  is the mapped phase error at pixel  $k$ .

The final objective is to obtain the unwrapped or absolute phase  $\varphi(k)$  from the noisy  $2\pi$  mapped phases  $\tilde{\varphi}(k)$  contained in the interferogram. At this step, we will be interested in the calculus of the phase difference from one pixel to the next

$$\tilde{\Delta}_\varphi(k) = [\tilde{\varphi}(k+1) - \tilde{\varphi}(k)]_{|2\pi} \quad (3)$$

that yields

$$\tilde{\Delta}_\varphi(k) = \left[ \delta_\varphi(k) + [\tilde{e}_\varphi(k+1) - \tilde{e}_\varphi(k)]_{|2\pi} \right]_{|2\pi} \quad (4)$$

where  $\delta_\varphi(k)$  is the true discrete phase derivative and its modulus is always supposed to be smaller than  $\pi$  (i.e., there is no aliasing).

Finally, the unwrapped phase  $\varphi$  can be obtained through the following recursive expression:

$$\varphi(k+1) = \varphi(k) + \tilde{\Delta}_\varphi(k). \quad (5)$$

## III. PARTICLE FILTER

### A. Introduction

As mentioned earlier, most of the traditional PU methods suffer from the lack of filtering capacity. Kalman-filter-based methods provide filtering capacity, but they are subject to linear and Gaussian constraints. The EKF is suitable to address the linear restriction, provided that the model presents weak nonlinearities which can be linearized in a proper manner. Instead, if the model has discontinuous nonlinearities, the EKF should not be employed [26].

In the case of PU, the observation model is nonlinear with non-Gaussian statistics. Moreover, if the unwrapped phase is chosen as state variable (i.e., we employ a 1-D EKF), then the observation model is nonlinear and discontinuous (wrapped phase). In this case, the application of the EKF is not straightforward. The correct application of the EKF requires the 2-D formulation proposed in [16]–[21], where the observation model contains real and imaginary parts of the complex interferogram [21]. Thus, the model remains nonlinear but continuous. A more complete introduction to the use of this type of approaches for PU can be consulted in [21].

In contrast to EKF, the application of a PF solution is more straightforward, since it exhibits the advantage of using only one dimension per pixel (directly the phase), hence bringing down complexity to its simplest case, and it is not subject to any linear or Gaussian constraints.

### B. PF Formulation

Hereinafter, to be coherent with the notation of state-space methods, the unwrapped phase at pixel  $k$ ,  $\varphi(k)$ , will be referred to as the corresponding state  $x_k$  at pixel  $k$ .

The suboptimal solution of a PF [22] can be adopted, since the continuous state space can be divided into  $N$  cells or states,  $\{x_k^i; i = 1, \dots, N\}$ , which correspond to all possible values of the phase. The resulting grid must be sufficiently dense to get a good approximation to the continuous state space.

Conceptually, the PF is simply a solution that represents the posterior probability function (*pdf*)  $p(x_k|z_{1:k})$ , by a distribution of  $N_s$  particles, defined hereinafter, extracted from the state space. The objective is to calculate estimates of the state  $x_k$  based on the set of all available observations  $z_{1:k} = \{z_i, i = 1, \dots, k\}$  up to pixel  $k$ .

The particles are defined as the set of states whose weights are the highest for each step. These particles, or selected group of samples, and their weights become the starting point for the next step of the algorithm. The basic stages at each step of the algorithm are represented in Fig. 1.

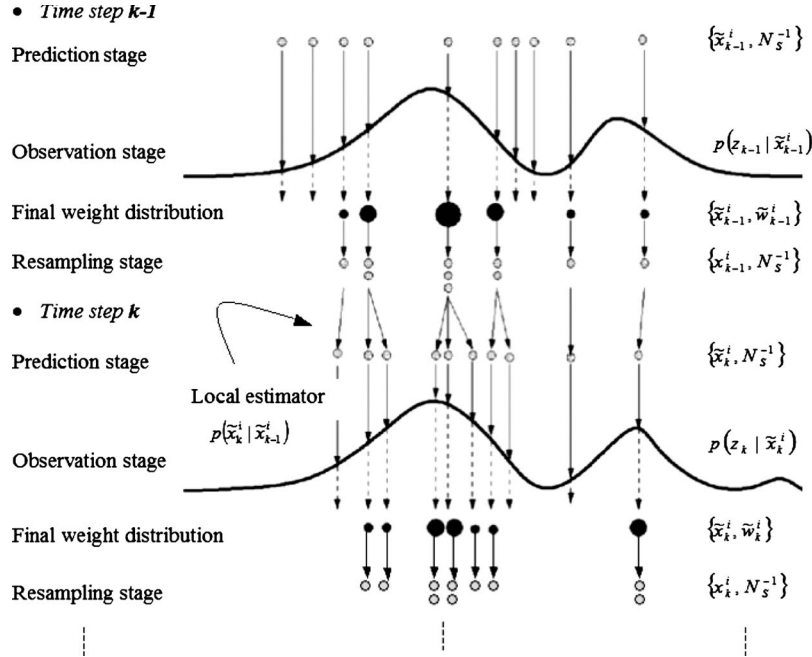


Fig. 1. Illustration of the stages of the PF algorithm for a pixel.

Considering the Markovian assumption, the *pdf*  $p(x_k|z_{1:k})$  can be obtained recursively from the *pdf*  $p(x_{k-1}|z_{1:k-1})$  calculated at previous pixel  $k - 1$ . It is supposed that the initial *pdf*  $p(x_0|z_0) \equiv p(x_0)$  is known. If it is not the case, the samples can be initially distributed uniformly over the state space. Next, at the present pixel  $k$ , the  $N_s$  samples are updated according to the action taken and the current observation  $z_k$ .

At first sight, this sequential problem could be divided into two stages: *prediction* and *update*. However, as it will be explained in the following, a third stage, named *resampling*, needs to be added. To broach prediction and update stages, it is assumed that evolution model and observation model exist and are known.

The prediction stage is solved by applying the evolution model  $p(x_k|x_{k-1})$  to each one of the  $N_s$  particles, generating a new set of samples. These samples represent the prediction of the state variable  $x_k$  without bearing in mind the observation at pixel  $k$ .

To consider the current observation  $z_k$ , the update stage is performed. The weights associated with each sample,  $w_k^i; i = 1, \dots, N_s$ , are obtained according to the observation model  $p(z_k|x_k)$ .

As it was introduced before, a third stage named *resampling* needs to be added in order to improve the performance of the filter. Let  $\{x_{0:k}^i, w_k^i\}_{i=1}^{N_s}$  denote a random measure to characterize the posterior *pdf*  $p(x_{0:k}|z_{1:k})$ , where  $\{x_{0:k}^i; I = 1, \dots, N_s\}$  is a set of support points with associated weights  $\{w_k^i; i = 1, \dots, N_s\}$ , and  $x_{0:k} = \{x_j; j = 0, \dots, k\}$  is the set of all states up to pixel  $k$ . The weights are normalized,  $\sum_i w_k^i = 1$ . Then, the posterior *pdf* at pixel  $k$  can be approximated as

$$p(x_{0:k}|z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(x_{0:k} - x_{0:k}^i) \quad (6)$$

where  $\delta(\cdot)$  represents the Dirac delta.

Let  $x_i, i = 1, \dots, N_s$  be samples easily generated from a function  $q(\cdot)$  called *importance density*. The weights are chosen by means of the *importance resampling* principle, which relies on the following. Suppose that  $p(x) \propto \pi(x)$  is a probability density from which it is not easy to draw samples but for which  $\pi(x)$  can be evaluated (as well as  $p(x)$  up to proportionality). In this context, a weighted approximation to  $p(\cdot)$  is

$$p(x) \approx \sum_{i=1}^{N_s} w^i \delta(x - x^i) \quad (7)$$

where

$$w_k^i \propto \frac{\pi(x^i)}{q(x^i)} \quad (8)$$

represents the normalized weight of the  $i$ th particle.

If the samples  $x_{0:k}^i$  are drawn by using an importance density, the weights are then defined by

$$w_k^i \propto \frac{p(x_{0:k}^i|z_{1:k})}{q(x_{0:k}^i|z_{1:k})}. \quad (9)$$

As presented in [22], if the importance density is chosen to factorize such that

$$q(x_{0:k}|z_{1:k}) = q(x_k|x_{0:k-1}, z_{1:k})q(x_{0:k-1}|z_{1:k-1}) \quad (10)$$

then samples  $x_{0:k}^i \sim q(x_{0:k}|z_{1:k})$  can be obtained by augmenting each of the existing samples  $x_{0:k-1}^i \sim q(x_{0:k-1}|z_{1:k-1})$  with the new state  $x_k^i \sim q(x_k|x_{0:k-1}, z_{1:k})$ .

Concerning  $p(x_{0:k}|z_{1:k})$ , after a few operations [22], it can be shown that

$$p(x_{0:k}|z_{1:k}) \propto p(z_k|x_k)p(x_k|x_{k-1})p(x_{0:k-1}|z_{1:k-1}). \quad (11)$$

Therefore, by substituting (10) and (11) into (9)

$$\begin{aligned} w_k^i &\propto \frac{p(z_k|x_k^i) p(x_k^i|x_{k-1}^i) p(x_{0:k-1}^i|z_{1:k-1})}{q(x_k^i|x_{0:k-1}^i, z_{1:k}) q(x_{0:k-1}^i|z_{1:k-1})} \\ &= w_{k-1}^i \frac{p(z_k|x_k^i) p(x_k^i|x_{k-1}^i)}{q(x_k^i|x_{0:k-1}^i, z_{1:k})}. \end{aligned} \quad (12)$$

Furthermore, if we consider that  $q(x_k|x_{0:k-1}, z_{1:k}) = q(x_k|x_{k-1}, z_k)$ , then only  $x_k^i$  and the current observation  $z_k$  need to be stored. The weight is then

$$w_k^i \propto w_{k-1}^i \frac{p(z_k|x_k^i) p(x_k^i|x_{k-1}^i)}{q(x_k^i|x_{k-1}^i, z_k)}. \quad (13)$$

The posterior density of probability is given by

$$p(x_k|z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(x_k - x_k^i) \quad (14)$$

where the weights are defined in (13). It can be shown that the larger the  $N_s$ , the more (14) approaches the true *pdf*  $p(x_k|z_{1:k})$ .

Finally, it is convenient to choose the importance density to be

$$q(x_k|x_{k-1}^i, z_k) = p(x_k|x_{k-1}^i). \quad (15)$$

This selection simplifies the calculus of the weights as follows:

$$w_k^i \propto w_{k-1}^i p(z_k|x_k^i). \quad (16)$$

The graphical evolution of the samples at each stage of a standard PF algorithm is also shown in Fig. 1.

It is assumed that the samples estimate only one parameter, distributed over the horizontal axis. The samples are represented by circles centered on the value of the parameter that they represent. The area of the circles represents the weight of the corresponding sample. In this example, the algorithm starts at pixel  $k-1$  with an unweighted distribution  $\{\tilde{x}_{k-1}^i, N_s^{-1}\}$ . It provides an approximation of  $p(x_{k-1}|z_{1:k-2})$ . For each particle, its associated weight is computed by using the information provided by the observation model  $p(z_{k-1}|\tilde{x}_{k-1}^i)$  at pixel  $k-1$ . It results in the weighted measure  $\{\tilde{x}_{k-1}^1, \tilde{w}_{k-1}^1\}$ , which yields an approximation of  $p(x_{k-1}|z_{1:k-1})$ . Finally, the resampling step, which will be explained hereinafter, selects only the "fittest" particles to obtain the unweighted measure  $\{x_{k-1}^1, N_s^{-1}\}$ , which is still an approximation of  $p(x_{k-1}|z_{1:k-1})$ . The next step of the algorithm starts with the evolution stage, and information from the local phase estimator  $p(\tilde{x}_k^1|\tilde{x}_{k-1}^1)$  is added at this moment. The evolution stage introduces a prediction and its uncertainty, yielding the measure  $\{\tilde{x}_k^1, N_s^{-1}\}$ , which is an approximation of  $p(x_k|z_{1:k-1})$ . The sequence of the algorithm is the same for the rest of subsequent pixels.

An *a priori* drawback of PF methods could be the constraint of a finite state space, but this is not the case in the interferometric PU problem. The PF solution that we propose here makes

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[ $\{x_k^i, w_k^i\}_{i=1}^{N_s}$ ] = PF [ $\{x_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, z_k$ ]
FOR  $i = 1, \dots, N_s$ :
  - Draw  $x_k^i \sim q(x_k|x_{k-1}^i, z_k)$ 
  - Assign the particle a weight,  $w_k^i$ , according to (13)
END FOR
- Calculate total weight  $t = SUM [\{w_k^i\}_{i=1}^{N_s}]$ 

FOR  $i = 1, \dots, N_s$ :
  - Normalize  $w_k^i = w_k^i / t$ 
END FOR

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Fig. 2. Pseudocode of a generic PF.

use of a sliding window  $W_k^s$  to cover the complete state space. This window is  $2\pi$  wide and is centered in the previous value for the unwrapped phase, given by  $PF_{\max}$ , introduced in the following. It is based on the assumption that the phase difference between two contiguous pixels must be contained in the interval  $(-\pi, +\pi]$ . In this way, this *sliding state space* allows us to afford the interferometric PU problem independently of the width of the complete state space. Note that computational cost increases linearly with  $N_s$ .

A common and critical problem affecting PFs is the degeneracy phenomenon. It means that after a few steps of the algorithm, all but one particle will have negligible weight. This problem implies that a considerable computational work will be dedicated to updating particles whose contribution to the approximation of the posterior (*pdf*)  $p(x_k|z_{1:k})$  is nearly zero. Therefore, degeneracy is an undesirable issue that should be avoided. The first idea to remove this effect could be the brute force, i.e., the use of a very large  $N_s$ , which is normally infeasible and inefficient. However, there exist some other solutions, like resampling or good choice of the importance density [22]. Resampling will be the one to be used with the PF presented in this paper due to its simplicity and excellent results. More details of the resampling are provided in next section. The pseudocode of a generic PF is shown in Fig. 2. Note that the sliding window update and resampling stages have been omitted and will be introduced as part of the PU algorithm.

### C. Resampling

The pseudocode of the resampling algorithm is shown in Fig. 3. This method allows the reduction of the effects of degeneracy. The basic idea of resampling is to drop particles that have small weights and to concentrate on those with large weights. A new set of samples is generated by resampling the set of samples and taking out, with replacement,  $N_s$  samples from the current set, proportionally to their weights. In this new set, for instance, the samples with the lowest probabilities will disappear. Next, the weights associated with the samples are scaled in order to represent the probability associated with each sample. The resulting set of samples is in fact an i.i.d. sample from the discrete posterior probability function  $p(x_k|z_{1:k})$ . Therefore, the weights can now be reset to  $w_k^i = 1/N_s$ .

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 $\left\{x_k^{j*}, w_k^j\right\}_{j=1}^{N_s} = \text{Resample} \left[ \left\{x_k^i, w_k^i\right\}_{i=1}^{N_s} \right]$ 
- Initialize the CWS*:  $c_i = 0$ 
FOR  $i = 2, \dots, N_s$ 
  - Build CWS:  $c_i = c_{i-1} + w_k^i$ 
END FOR
- Start at the bottom of CWS:  $i = 1$ 
- Draw a starting point:  $u_i \sim U[0, 1/N_s]$ 
FOR  $j = 1, \dots, N_s$ 
  - Move along the CWS:  $u_j = u_i + (j-1) \cdot (1/N_s)$ 
  WHILE  $u_j > c_i$ 
     $i = i + 1$ 
  END WHILE
  - Assign sample:  $x_k^j = x_k^i$ 
  - Assign weight:  $w_k^j = 1/N_s$ 
END FOR
* Cumulative Weight Sum (CWS) vector

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Fig. 3. Pseudocode of the resampling algorithm.

#### IV. PARTICLE FILTER PHASE UNWRAPPING

##### A. Introduction

The unidimensional PU problem will be addressed first for a better process understanding. In this case, the probabilities  $p(x_k^i | x_{k-1}^i)$  and  $p(z_k | x_k^i)$  in (13) are defined by the evolution and observation models at pixel  $k$ , given by

$$x_k^i = x_{k-1}^i + \Delta \hat{\varphi}_{k-1} + n_{k-1} \quad (17)$$

$$z_k = x_k^i + v_k \quad (18)$$

where  $\Delta \hat{\varphi}_{k-1}$  represents a local phase-slope estimate, while  $n_k$  and  $v_k$  are supposed to be approximate models for the noise present in both evolution and observation stages, respectively.

Several techniques can be used to obtain an estimation of the phase slope. For instance, the mode of the power spectral density was used in [16] and [17] due to its supposed insensitiveness and robustness to superimposed white noise, whereas a phase-average-based method was used in [27], and a matrix pencil approach was proposed in [28]. All of them are perfectly applicable. The first one has been selected in this case for the sake of comparison.

It is well known that the estimated covariance matrix for correlated SAR images is statistically described by the complex Wishart distribution, with the distribution of the interferometric phase being a marginal density distribution of the Wishart one [29], [30]. Therefore, noise  $v_k$  has been modeled as a discrete marginal Wishart distribution. It is important to note that this modelization is not strictly necessary, since this approach can work with any distribution or probability density function, known or not, thus providing independence from noise statistics. When the statistics is known *a priori*, the algorithm can use its analytical expression, thus making the computations easier. When the statistics is not available, the algorithm can calculate it.

Concerning  $n_k$ , it models the uncertainty introduced by the local phase estimator (based on the power spectral density

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- Initial weight distribution:  $w_{i=1, \dots, N_s}^o$ . An initial state  $x_o$  is selected.

FOR  $k = 1, \dots, T$

- Phase slope estimations  $\Delta \hat{\varphi}_H$  and  $\Delta \hat{\varphi}_V$

$$\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s} = PF \left[ \left\{x_{k-1}^i, w_{k-1}^i\right\}_{i=1}^{N_s}, z_k \right]$$

- Unwrapped phase:

- Cell whose weight is maximum:  $x_k = x_k^i \big|_{\max(w_k^i)}$

- Sliding window update:  $\bar{W}_k^s$

- Resample particles using algorithm *Resampling*:

$$* \left\{x_k^{j*}, w_k^j\right\}_{j=1}^{N_s} = \text{Resample} \left[ \left\{x_k^i, w_k^i\right\}_{i=1}^{N_s} \right]$$

END FOR

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Fig. 4. Pseudocode of the proposed 2-D PU algorithm using a PF (PFPU).

in this paper). It has been modeled as a Gaussian discrete distribution for the sake of comparison with [16]–[21].

##### B. PFPU Algorithm

To apply the PFPU to 2-D interferograms, any prediction estimate will be calculated, depending on two neighbors. Hence, the stage concerning the evolution model (17) needs to be modified as follows:

$$x_k^i \sim \frac{1}{2} \cdot [p_H(x_k^i | x_{k-1}^i) + p_V(x_k^i | x_{k-m}^i)] \quad (19)$$

where  $p_H(x_k | x_{k-1}^i)$  is the evolution model in the horizontal direction at pixel  $k - 1$ , given by

$$x_k^i = x_{k-1}^i + \Delta \hat{\varphi}_{k-1}^H + n_{k-1}^H \quad (20)$$

where  $x_{k-1}^i$  refers to the unwrapped phase at pixel  $k - 1$ ,  $\Delta \hat{\varphi}_{k-1}^H$  refers to the horizontal local phase estimation at pixel  $k - 1$ , and  $n_{k-1}^H$  refers to the Gaussian noise sample at pixel  $k - 1$  that models the uncertainty introduced by the local phase estimator. Note that pixel  $k - 1$  is the previously unwrapped one, placed in the same row and adjacent to the current pixel  $k$ .

On the other hand,  $p_V(x_k | x_{k-m}^i)$  is the evolution model in the vertical direction, at pixel  $k - m$ , given by

$$x_k^i = x_{k-m}^i + \Delta \hat{\varphi}_{k-m}^V + r_{k-m}^V \quad (21)$$

where  $x_{k-m}^i$  refers to the unwrapped phase at pixel  $k - m$ ,  $\Delta \hat{\varphi}_{k-m}^V$  refers to the vertical local phase estimation at pixel  $k - m$ , and  $r_{k-m}^V$  refers to the Gaussian noise sample at pixel  $k - m$  that models the uncertainty introduced by the local phase estimator. Note that pixel  $k - m$  is the previously unwrapped one, placed in the same column and adjacent to the current pixel  $k$ .

The pseudocode of the 2-D PFPU algorithm is shown in Fig. 4.

##### C. 2PFPU Algorithm

As it will be shown later with some examples, the row-by-row PFPU solution presented in the previous section (see Fig. 5)



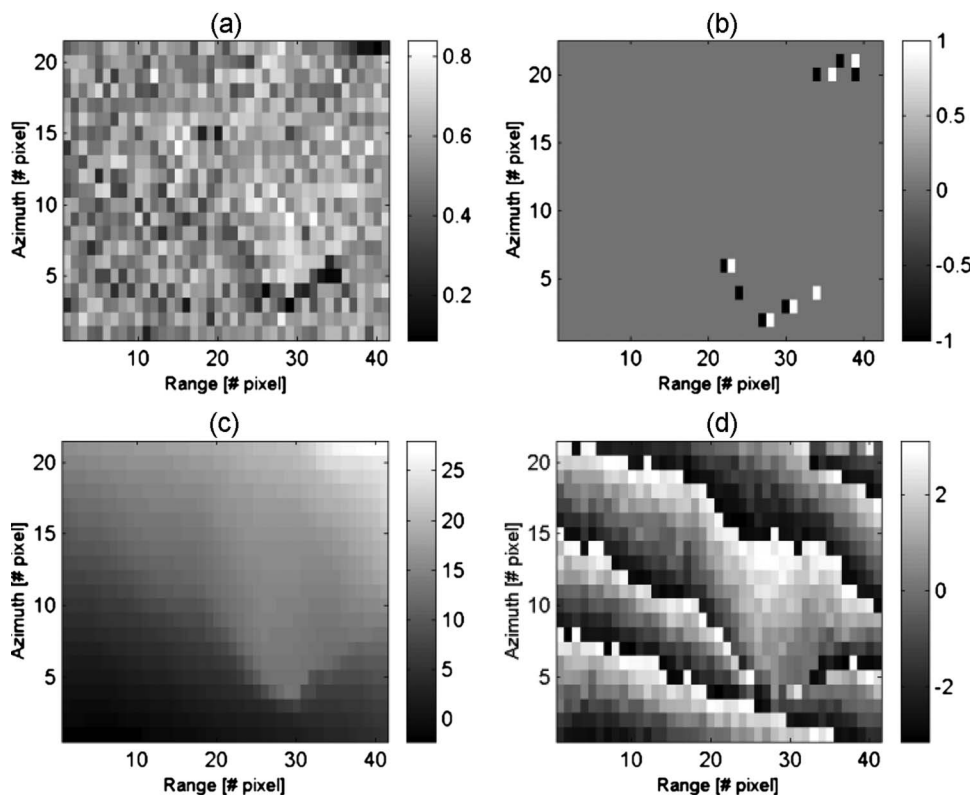


Fig. 8. Longs Peak synthetic interferogram. (a) Coherence. (b) Residues. (c) True absolute unwrapped phase. (d) Noisy input signal (wrapped phase).

## V. RESULTS

The results in this section illustrate the performance of different PU solutions as well as the influence of the noisy input signal. The following PU solutions are compared: the branch-cut algorithm proposed by Goldstein [1], Flynn's algorithm [1], the PCG [1], the EKF algorithm described in [16]–[21], two approaches of the GbF proposed in [23] and [24], the row-by-row PF solution (PFPU) proposed in this paper (see Figs. 4 and 5), and the improved 2PFPU algorithm. The two GbF options consist of selecting the state whose weight is the maximum  $GbF_{\max}$  or selecting the average state of the distribution  $GbF_{\text{av}}$ . Algorithms have been applied to different synthetic and real scenarios. Note that all the phase results are presented in radians.

1) *Synthetic Data Over Longs Peak (Colorado) [1]*: We have applied all, but the 2PFPU algorithm, to one example that most of the methods introduced in [1] fail to unwrap: a portion of a synthetic interferogram from a mountainous terrain around Longs Peak, Colorado (Fig. 8). The data set, as detailed in [1], was generated with a high-fidelity InSAR simulator by employing digital terrain elevation models from the U.S. Geological Survey. We have also chosen this synthetic example because the true unwrapped phase is available, so one can compare it with the phase delivered by the PU algorithms.

In the study area, there exist zones where the coherence (i.e., the quality) of the data is very poor. For instance, in the surroundings of rows 3–5 and columns 25–35, it reaches values lower than 0.3 [see Fig. 8(a)]. As a consequence, residues appear [see Fig. 8(b)]. We will illustrate the following examples by showing the final unwrapped phase provided by the PU

algorithms, which can be compared against the true absolute unwrapped phase represented in Fig. 8(c). In addition, the result of wrapping the solution (rewrapped phase) will be analyzed for detecting potential errors introduced by the PU algorithms with respect to the original wrapped phase (i.e., the input signal), which is shown in Fig. 8(d).

The PU algorithm proposed by Goldstein [1] identifies the residues and defines the so-called branch cuts between them, which will not be crossed by the unwrapping paths. This method attempts to minimize the branch-cut lengths. As a result, many areas can be completely isolated and, consequently, not unwrapped consistently with the rest of the interferogram. This is the case shown in Fig. 9.

The results obtained by applying Flynn's algorithm and the PCG method [1] are shown in Figs. 10 and 11, respectively. It can be observed how both methods also fail to unwrap correctly this synthetic data set.

As observed in Fig. 12, the EKF [16]–[21] fails as a consequence of a concentration of residues around the pixel (3, 27). It produces errors in the slope estimates and in the unwrapped phase in this area, thus generating an error which propagates in a diagonal fashion and producing an erroneous phase discontinuity.

In contrast, as shown in Fig. 13 for the same test interferogram, the PFPU solution does not suffer from this error propagation, since the PF is able to fix this problem. Although more expensive than other techniques, the computational cost of the PFPU was only a few seconds to unwrap this area, which is much shorter than the analogous GbF solutions as a consequence of using particles instead of a complete grid of states.

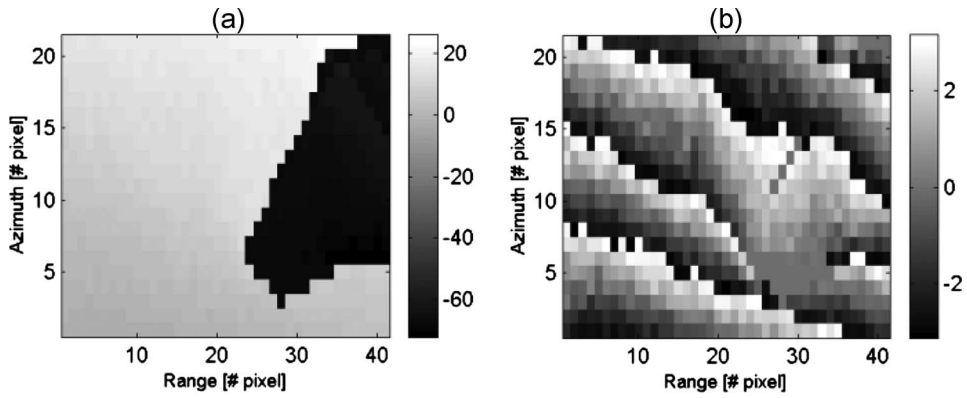


Fig. 9. Goldstein solution for Longs Peak synthetic interferogram. (a) Unwrapped phase. (b) Rewrapped phase.

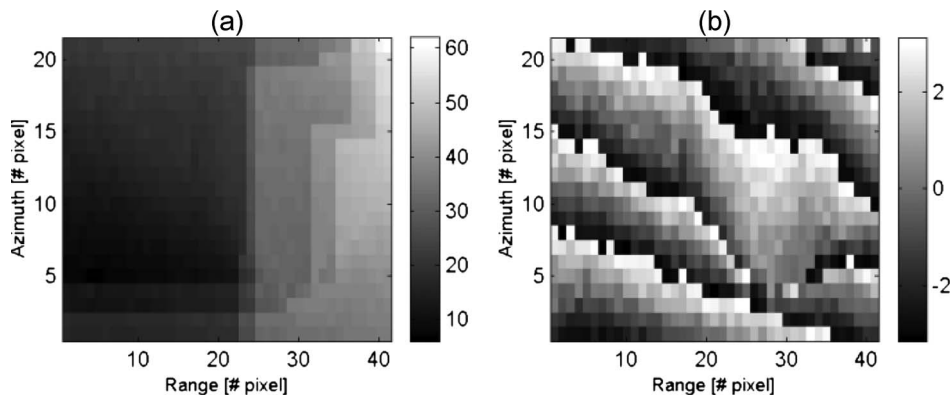


Fig. 10. Flynn solution for Longs Peak synthetic interferogram. (a) Unwrapped phase. (b) Rewrapped phase.

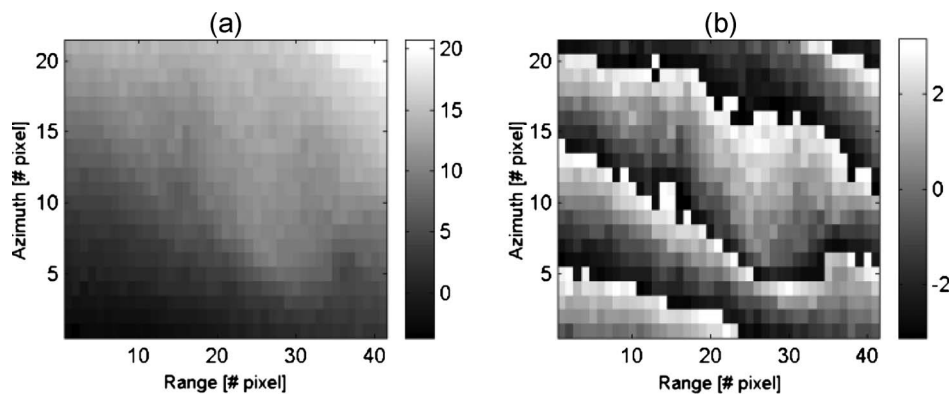


Fig. 11. PCG solution for Longs Peak synthetic interferogram. (a) Unwrapped phase. (b) Rewrapped phase.

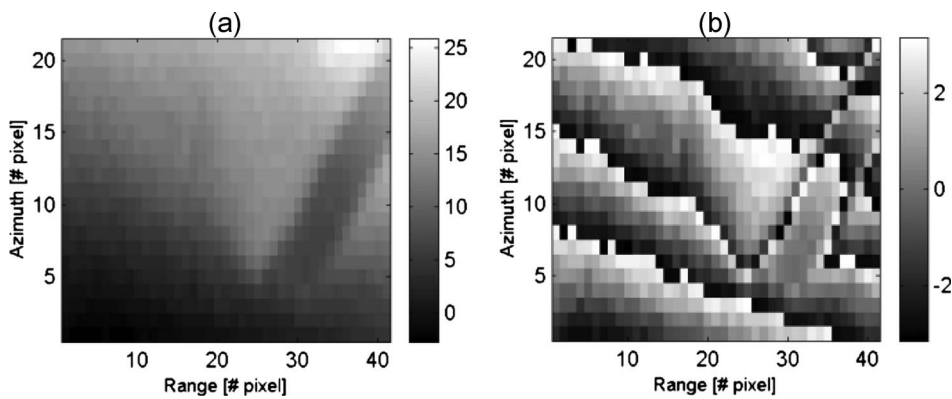


Fig. 12. EKF solution for Longs Peak synthetic interferogram. (a) Unwrapped phase. (b) Rewrapped phase.



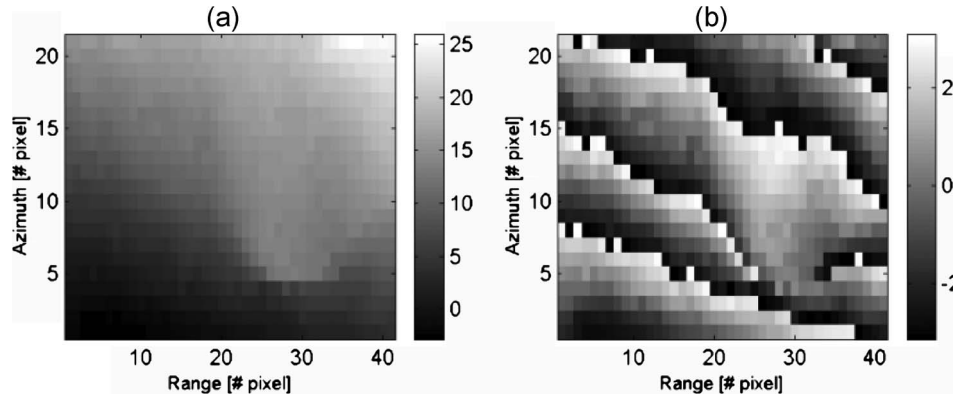


Fig. 13. PFPUsolution for Longs Peak synthetic interferogram. (a)  $PFPU_{\text{max}}$  unwrapped phase. (b)  $PFPU_{\text{max}}$  rewrapped phase. The continuous space contained inside the  $2\pi$  sliding window has been translated into a discrete state space composed of  $N = 100$  cells, and a maximum of  $N_s = 50$  particles will be used.

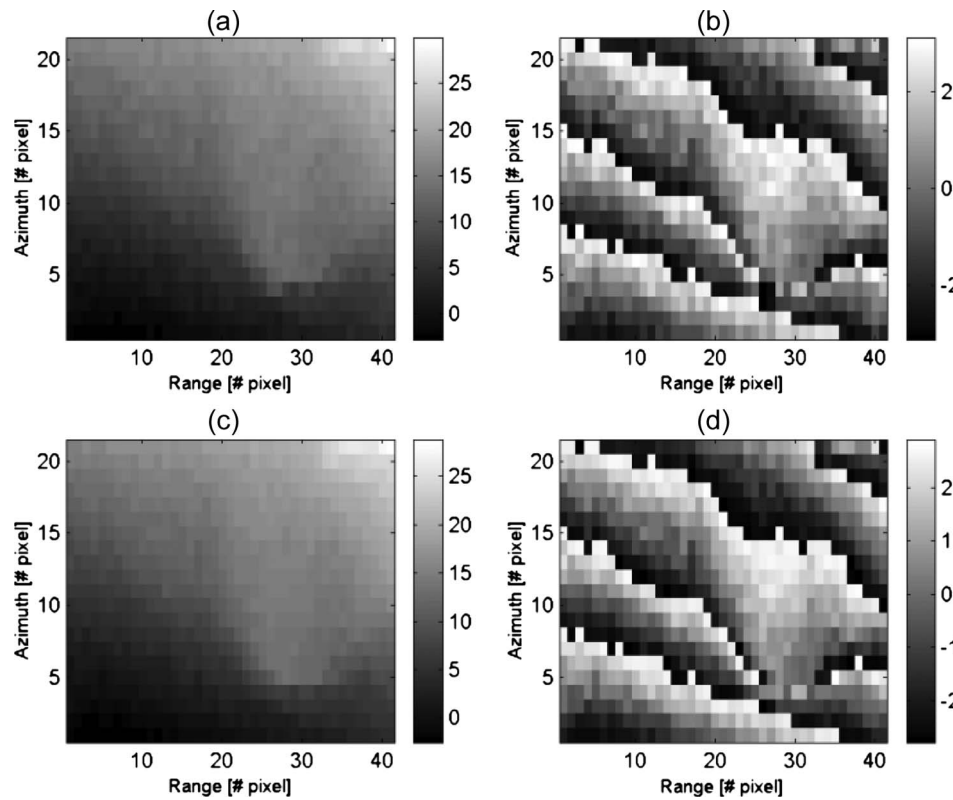


Fig. 14. GbF solution for Longs Peak synthetic interferogram. (a)  $GbF_{\text{max}}$  unwrapped phase. (b)  $GbF_{\text{max}}$  rewrapped phase. (c)  $GbF_{\text{av}}$  unwrapped phase. (d)  $GbF_{\text{av}}$  rewrapped phase. The continuous space contained inside the  $2\pi$  sliding window has been translated into a discrete state space composed of  $N = 100$  cells.

The results provided by both GbF solutions,  $GbF_{\text{max}}$  and  $GbF_{\text{av}}$ , are also shown in Fig. 14. As expected, the results are quite similar to the ones obtained by the PFPUsolution, but they have been obtained with a 35% higher computational effort.

2) *Small Area of a Real ERS-1 and ERS-2 Tandem Interferogram Over Alicante Province:* The following results correspond to a small crop extracted from a real interferogram obtained with images acquired by the ERS-1 and ERS-2 satellites in tandem configuration. The same algorithms as in the previous example have been compared. Coherence and input wrapped phase are shown in Fig. 15. As observed in Fig. 16, the EKF fails in two different areas. First, a concentration of residues around the pixel (40, 80) produces errors in the slope estimates and in the unwrapped phase in this area. Second,

the conjunction of low coherence and high variance of the local phase estimators in pixel (7, 70) produces an error which propagates in a diagonal fashion, generating an erroneous phase discontinuity.

For this example, the rest of tested algorithms show similar results (see Figs. 17–20). The main differences appear for Goldstein’s and the PCG solution. Goldstein’s algorithm failed to unwrap a small group of pixels around (40, 80), since they appear completely isolated as a result of the combination of the branch cuts [see Fig. 17(a)]. In the case of PCG, by inspecting the rewrapped phase in Fig. 19(b), it can be observed that part of a phase cycle has been lost for a group of pixels around (40, 80). This problem is common in minimum-norm methods. On the other hand, the solution provided by the PFPUsolution

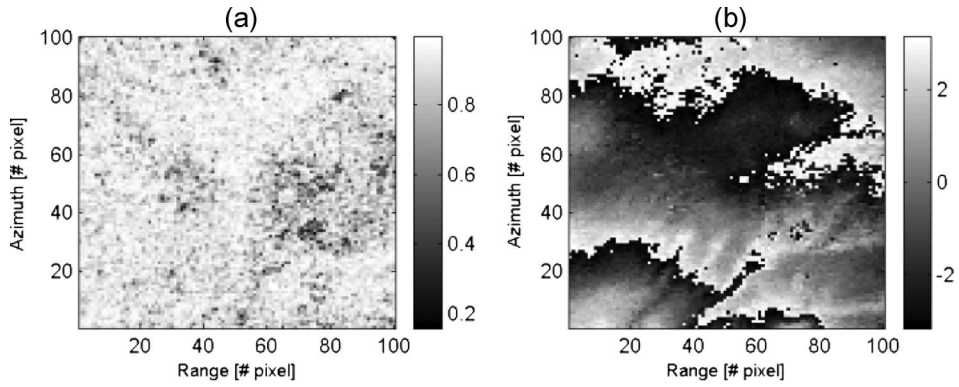


Fig. 15. Small real interferogram with ERS images. (a) Coherence. (b) Input wrapped phase.

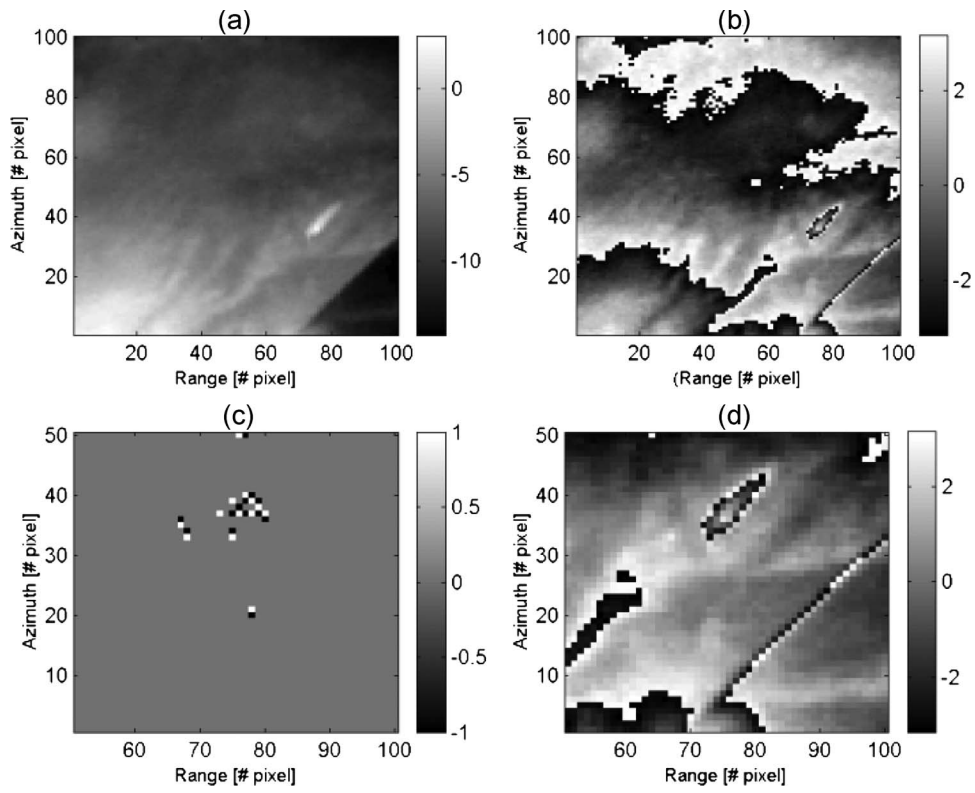


Fig. 16. EKF solution for the small ERS interferogram. (a) Unwrapped phase. (b) Rewrapped phase. (c) Zoom of residues. (d) Zoom of rewrapped phase.

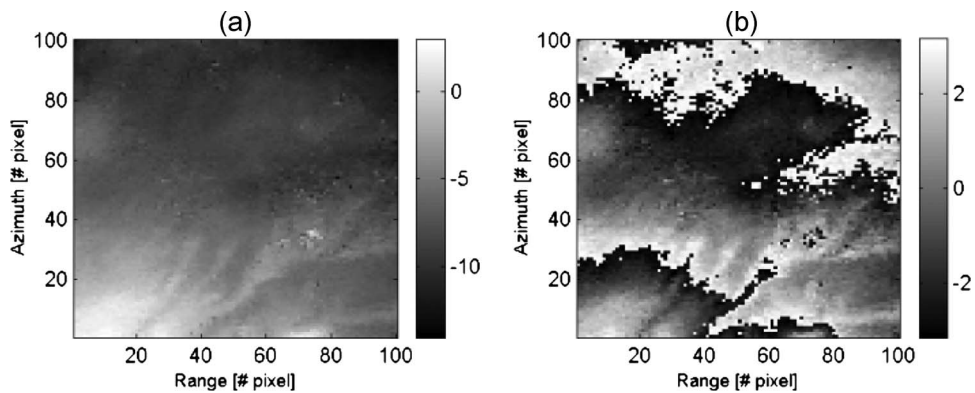


Fig. 17. Goldstein solution for the small ERS interferogram. (a) Unwrapped phase. (b) Rewrapped phase.

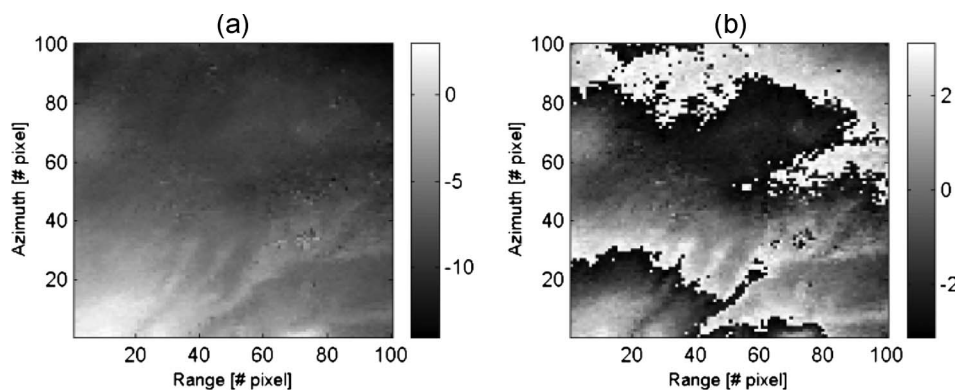


Fig. 18. Flynn solution for the small ERS interferogram. (a) Unwrapped phase. (b) Rewrapped phase.

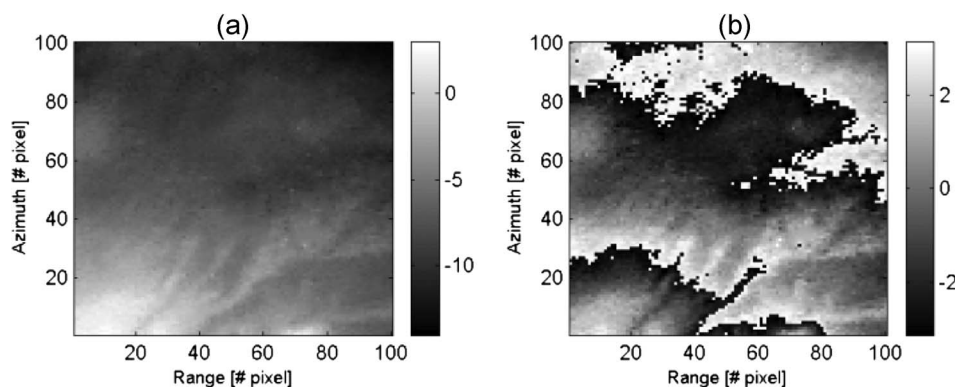


Fig. 19. PCG solution for the small ERS interferogram. (a) Unwrapped phase. (b) Rewrapped phase.

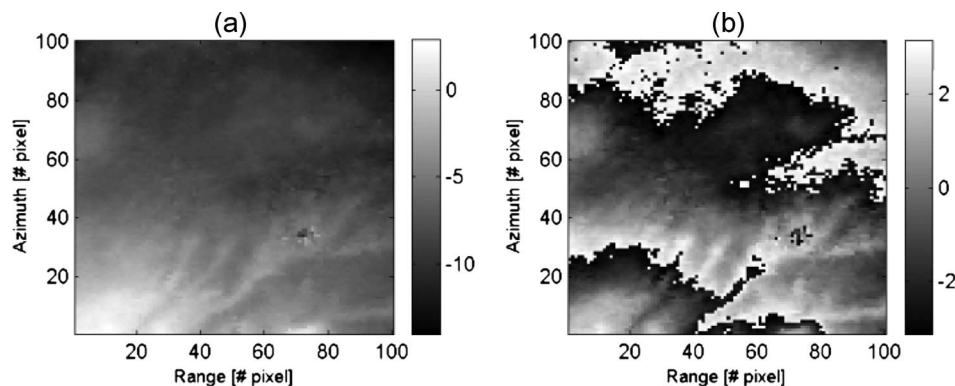


Fig. 20. PFPU solution ( $PFPU_{\max}$ ) for the small ERS interferogram. (a) Unwrapped phase. (b) Rewrapped phase. The continuous space contained inside the  $2\pi$  sliding window has been translated into a discrete state space composed of  $N = 100$  cells, and a maximum of  $N_s = 50$  particles will be used.

is smoother than that of Goldstein and Flynn solutions in the upper part of the interferogram due to the filtering capability of this approach.

3) *X-SAR Interferogram Over Mount Etna*: A simple row-by-row PF algorithm (PFPU) has been used so far for computational cost reasons. It has been shown how it can deal with residues and even recover from errors. However, there exist many situations in which zones of very low coherence or with a high concentration of residues are present. In these cases, it would be preferable to unwrap them the last. To do so, an enhanced algorithm (2PFPU) has been introduced in Section IV-C, combining a path-following technique with the PF solution. The evident drawback of this method is that the computational cost grows from the necessity of storing

the particle distribution at each unwrapped pixel that still has any unwrapped neighbors. However, filtering and PU capacities are more powerful than in the row-by-row approach, since the phase is unwrapped moving from the higher quality zones to the lower ones. Just to give an approximate comparison about the relative computational cost of these approaches, it took only a few seconds to process the interferogram by using the Goldstein algorithm, 1 min by using the PCG, nearly half an hour by means of Flynn's algorithm, and nearly 8 h by means of the 2PFPU method.

The data shown in Figs. 21 and 22 are the coherence and input wrapped phase, respectively, extracted from a real interferogram formed by a pair of SLC images with  $1718 \times 1992$  pixels, acquired by the X-SAR mission over Mount Etna. There exist

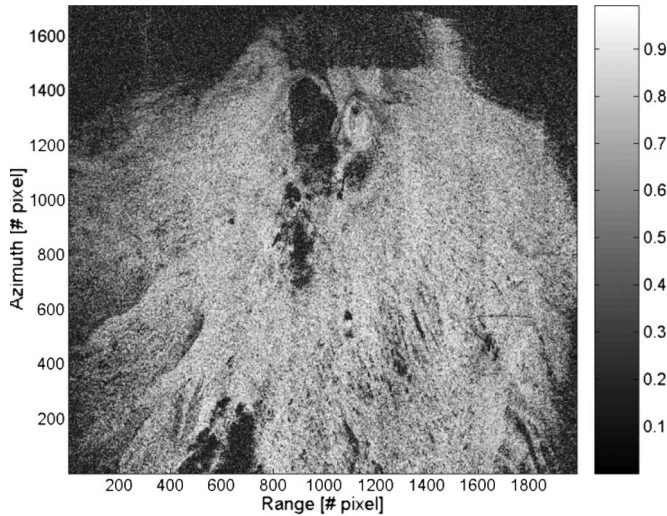


Fig. 21. Coherence of the X-SAR interferogram over Mount Etna.

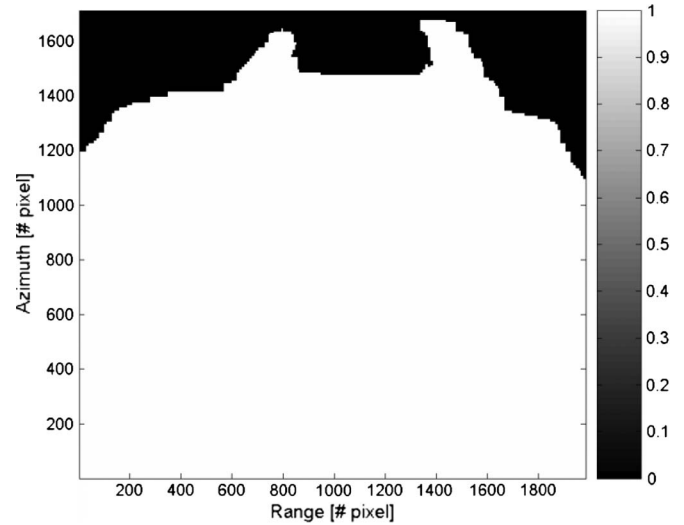


Fig. 23. Mask employed for unwrapping the X-SAR interferogram over Mount Etna. Black areas will not be unwrapped.

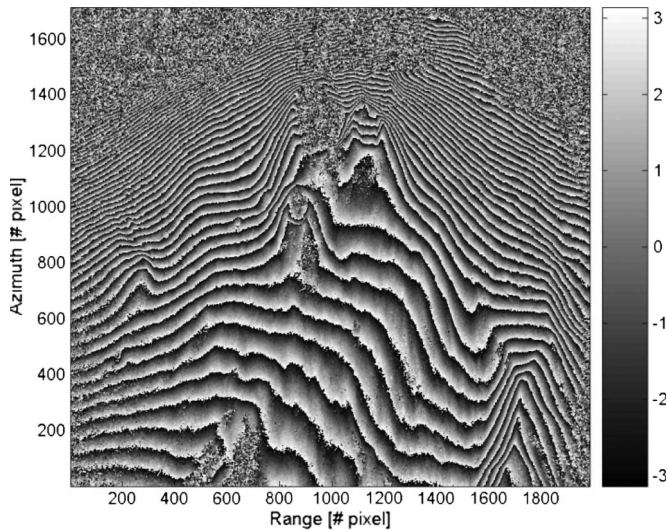


Fig. 22. Input wrapped phase of the X-SAR interferogram over Mount Etna.

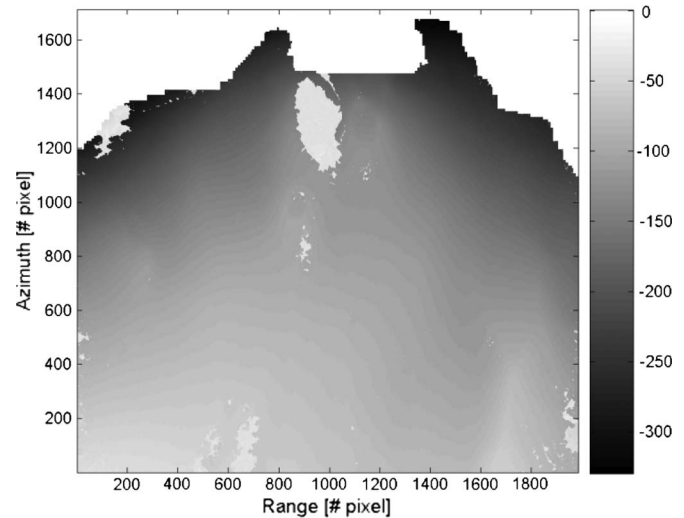


Fig. 24. Goldstein solution for the X-SAR interferogram over Mount Etna: Unwrapped phase.

some areas where the phase information is incoherent, one of them corresponding to the sea (at the top) and others probably due to the presence of vegetation and to shadow areas (as in the volcano crater). We have masked out the sea region but not the other low-quality areas. The mask employed in the subsequent unwrapping is shown in Fig. 23. Black areas will not be unwrapped.

The 2PFPU algorithm will be compared to Goldstein and Flynn solutions, since they share the path-following concept, and also to the PCG solution, as a comparison with minimum-norm methods. In some cases, an additional test has been included here. The difference between the unwrapped phase by each algorithm and the original interferogram has been computed, and the result has been wrapped again. This process should result in a noisy constant, and no bias should appear even for large-size images.

Traditional path-following techniques, Goldstein and Flynn (see Figs. 24 and 25), fail partially to unwrap this interferogram because there appear some isolated regions in the final solution.

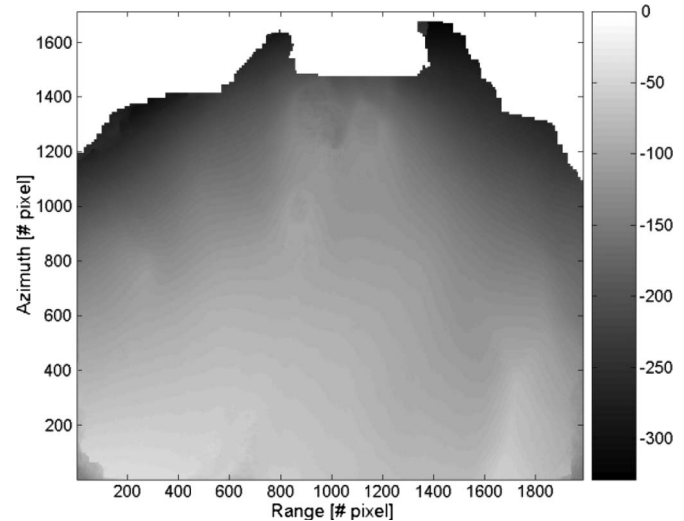


Fig. 25. Flynn solution for the X-SAR interferogram over Mount Etna: Unwrapped phase.

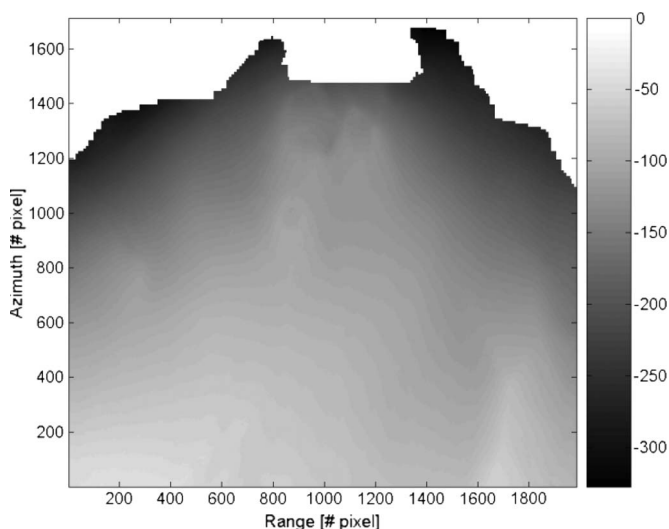


Fig. 26. PCG solution for the X-SAR interferogram over Mount Etna: Unwrapped phase.

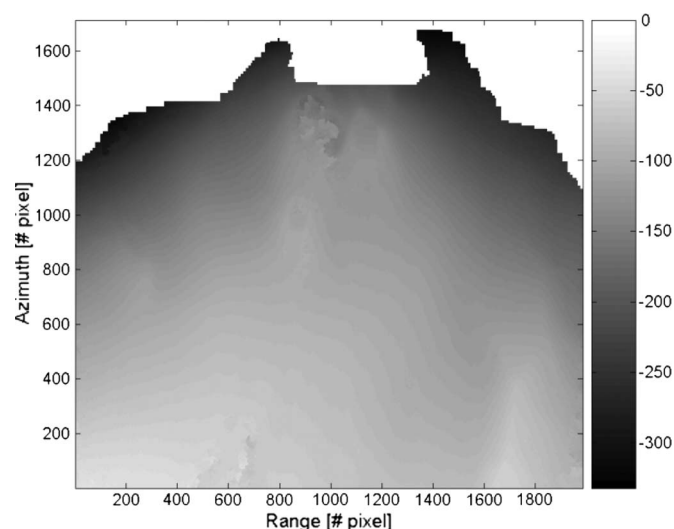


Fig. 28. 2PFPU solution for the X-SAR interferogram over Mount Etna: Unwrapped phase. The continuous space contained inside the  $2\pi$  sliding window has been translated into a discrete state space composed of  $N = 100$  cells, and a maximum of  $N_s = 50$  particles will be used.

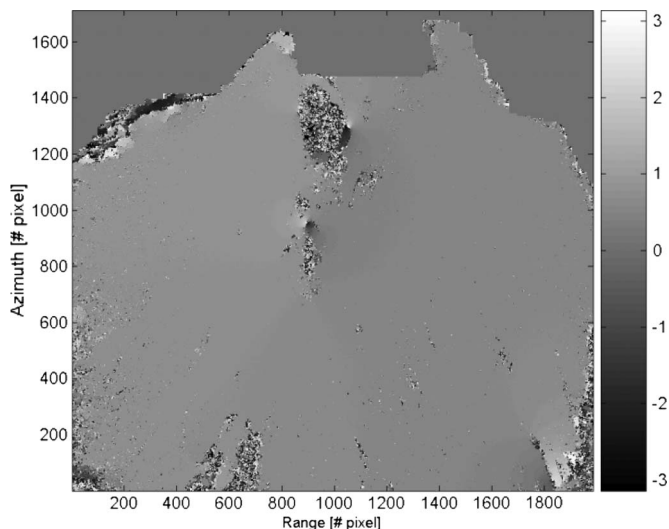


Fig. 27. PCG solution for the X-SAR interferogram over Mount Etna: Wrapped difference between the unwrapped phase and the input wrapped phase.

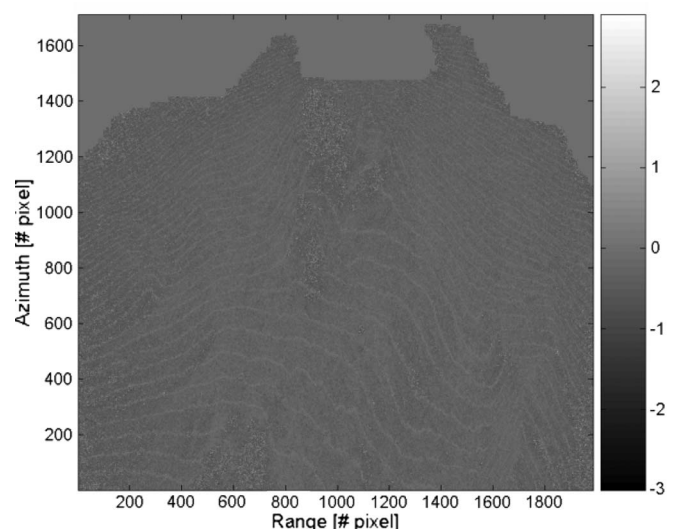


Fig. 29. 2PFPU solution for the X-SAR interferogram over Mount Etna: Wrapped difference between the unwrapped phase and the input wrapped phase.

On the other hand, the PCG approach results in a smooth solution (see Fig. 26) without isolated regions, as expected. In this case, the quality of the solution can be tested by observing the difference of the unwrapped and the original phase, which is shown in Fig. 27. Other than the expected noisy areas, there appear several discontinuities or fringes in this image, so the final solution is not satisfactory.

Finally, Figs. 28 and 29 show the capacities for the enhanced algorithm 2PFPU, combining a path-following technique with a PF and the omnidirectional phase estimator. Fig. 28 shows the unwrapped phase. The quality of this solution is shown by Fig. 29, since the wrapped difference between the solution and the original interferogram exhibits a noisy constant aspect, and no bias has appeared even for this large interferogram. Note that the noisy aspect is due to the filtering incorporated in this technique.

## VI. CONCLUSION

A new solution that simultaneously filters and unwraps the phase contained in an interferogram is presented in this paper.

This solution is based on a PF. Two variants of this method (PFPU and 2PFPU) have been implemented: The first one is a simple row-by-row unwrapping algorithm, and the second one is the result of combining a PF with AI search strategies and an enhanced version of the local phase estimator introduced in [16]–[21]. These algorithms have been compared to some conventional methods like the branch-cut algorithm proposed by Goldstein, Flynn’s algorithm, and PCG. This solution performs better in some cases, since PF can deal with zones containing residues.

PF-based solutions have also been compared to the EKF method and the GbF algorithm, which share the same

philosophy of simultaneously filtering and unwrapping. For some situations which the EKF fails to unwrap, a PF solution has shown a better performance and the capacity to recover from errors. It has also been shown that PF solutions perform better than the grid-based ones, since they obtain the same results with the advantage of a lower computational cost.

Our research lines at present and in the near future are focused on the following two main issues: first, utilization of other local phase estimators, for instance, the matrix pencil approach proposed in [28], and second, the combination of the 2PFPU approach with region-growing techniques will be analyzed, since a better performance is expected from the synergy between both strategies.

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